Optimization of Multilayer Antireflection Coatings Using an Optimal Control Method

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Abstract—**The design of a thin, light weight and broadband radar absorber is a problem of considerable interest and is cast in this paper as a minimization problem of the following quantities, namely the reflection coefficient at the set of frequencies $\{f_1, f_2, \cdots, f_n\}$ and the thickness (or surface mass) of the absorber. We attempt to synthetize an absorber with a undefined number of layers and assume we have the freedom to choose the permittivity and the permeability of the material in each layer from a set of m specified value of $\epsilon_k(f)$ and $\mu_k(f)$. The usual approach to the design problem is to consider classical types of absorbers, such as Dallenbach or Jaumann layers [1]. In this paper, we present a design procedure based upon an Optimal Control method, that simultaneously determines both the material properties of the different layers as well as their thicknesses, to minimize at the same time the reflection coefficient over a prescribed range of frequencies and surface mass or thickness. Illustrative examples of multilayer absorbers synthetized with this method are presented and the results are compared with those designed by using the Simulated Annealing method.**

I. Introduction

IN THIS paper we consider the problem of synthetizing a multilayer Radar Absorbing Material (RAM) coating for reducing the radar cross section (RCS) of radar targets using an Optimal Control approach. Typically, the specifications for an antireflection coating are that it should be lightweight, thin and have a broadband response over several octaves, e.g., 4 or 5.

The usual approach to designing such a coating is to use classical types of absorbing screens, such as Salisbury, graded index or Jaumann screens [1]. At optical frequencies, multilayer lossless dielectrics are often used as anti-reflection coatings. All these types of screens, except the last one, can be designed by following simple procedures. The Salisbury or graded index screens are typically designed using approximate closed form formulas, whereas an optimization scheme is employed for the design of Jaumann screens, as well as for optical coatings [2], [3]. Typically, the parameter space, e.g., range of materials, thickness or arrangement of the layers, etc., in which the optimization is carried out is fairly limited, and the design procedure is applicable only to a specific type of screen. In this paper, we attempt to overcome this limitation, by using a generalized synthesis procedure based upon an Optimal Control approach. This procedure systematically determines both the thickness and the material parameters of the layers that minimize at the same time the reflection coefficient over a specified range of frequencies and the total thickness or surface mass. Although we concentrate on the case of normal incidence in this paper, the method can be readily extended to the oblique incidence case, with some modifications (see Appendix).

The problem we wish to address can be stated as follows: using available materials with given permittivity $\epsilon(f)$ and permeability $\mu(f)$, design a multilayer screen with minimal reflection coefficients at a prescribed set of frequencies $\{f_1, f_2, \cdots, f_n\}$. The screen can be specified to have either minimal thickness or surface mass.

Consider a plane wave normally incident on a multilayer coating backed by a perfect electric conductor as shown in Fig. 1 (T is the total thickness of the layer). From Maxwell equations, we can readily derive:

$$j\omega\epsilon E_y(x,f) + \frac{dH_z}{dx} = 0$$

$$j\omega\mu H_z(x,f) + \frac{dE_y}{dx} = 0 \tag{1}$$

where $\omega = 2\pi f$ and f is the frequency of the incident wave.

Next, we use the usual definition for the wave impedance Z given by

$$Z(x) = \frac{E_y(x)}{H_z(x)} \tag{2}$$

By differentiating (2), using (1) and introducing the relative impedance Z_r, we can obtain the following equation:

$$\frac{dZ_r}{dx} = j\omega_r\epsilon_r Z_r^2 - j\omega_r\mu_r \tag{3}$$

where:

$$Z_r = Z/Z_0 = Z \times \sqrt{\epsilon_0/\mu_0}$$

$$\omega_r = \omega/c = w \times \sqrt{\epsilon_0\mu_0}$$

$$\epsilon_r = \epsilon/\epsilon_0 \quad \text{and} \quad \mu_r = \mu/\mu_0.$$

Manuscript received September 10, 1991; revised February 10, 1992.

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IEEE Log Number 9201718.

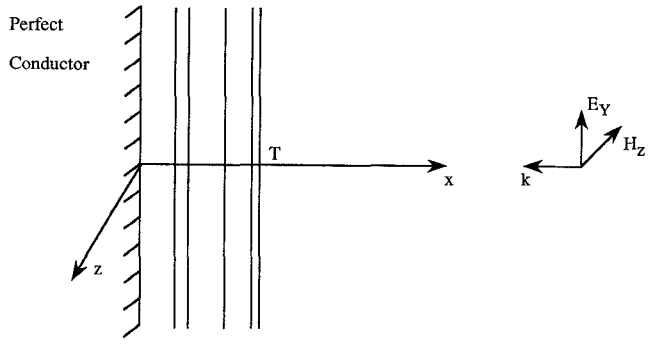


Fig. 1. Geometry of the problem.

In the following we shall drop the subscript r to simplify the notations. Thus, in the following, Z will stand for relative impedance, ϵ and μ for relative permittivity and permeability.

Equation (3) is valid for all point x in the thickness of the layer. It is recognized to be a Riccati equation, whose coefficients depend upon f, as does the impedance function Z. Next, we identify the parameters that are at our disposal for solving the optimization problem. They are:

(i) number of layers
(ii) material parameters of each layer
(iii) thickness of the layers

In contrast with the conventional approach, we do not a priori specify the number of unknown parameters to be dealt with in the optimization problem. This enables us to explore a wider range of design options than would be possible if the number of parameters were prespecified, as for instance, is the case for Jaumann screens.

We begin by observing that the boundary condition on the perfect electric conductor located at $x = 0$ requires that Z be equal to zero there. At the external surface of the layer, i.e., at $x = T$, we desire that Z be as close to $1 + j0$ as possible. Thus, we wish to transform the reflection coefficient given by $R = (Z - 1)/(Z + 1)$ from -1 to 0 by the introduction of the multilayer coating. Returning to (3), we can regard it as a nonlinear differential equation for Z with prescribed initial value $Z = 0$ at $x = 0$ and a desired final value of $Z = 1$ at $x = T$. We identify this as a problem of Optimal Control and use a minimal time control approach to derive the solution [4].

We begin by providing a brief review of the theory of the Optimal Control in minimal time and subsequently describe how this theory is applied to solve the problem at hand in Section II. Next, we further generalize the approach such that we can simultaneously minimize the reflection coefficient as well as the total thickness of the layer, or surface mass (see Section III). Finally, in Section IV we present some representative numerical results to illustrate the application of the method.

II. Optimal Control in Minimal Time

2.1 Brief Account of Optimal Control Procedure

In this section we briefly describe the basic procedure for the application of the optimal control method for the solution of the multilayer synthesis problem. The interested reader is referred to [5], [6] for further details.

We consider a dynamic process described by a system of n time differential equations, called the state equations:

$$\dot{Z}_i(t) = f_i\{Z_1(t), \cdots, Z_n(t); u_1(t), \cdots, u_m(t); t\} \quad (4)$$

$Z(t)$ is the state vector. $Z(t)$ belongs to $Z_{ad} \subset \mathbb{R}^n$.
$\dot{Z}$ is the derivative of Z with respect to time.
$u(t)$ is the control vector. $u(t)$ belongs to $U_{ad} \subset \mathbb{R}^m$.
Z_{ad} is the set of allowable states; U_{ad} is the set of allowable controls.

In following sub-sections, we will point out the correspondence between t, $Z(t)$ and $u(t)$, and the height in the screen x, impedance at height x: $Z(x)$ and the medium parameters respectively.

The optimal control problem is to take the system from a given initial state at $t = 0$, to a final state at the final time $t = T$ which is as close to the goal state as possible. The objective is to do this with a minimal cost, following the path from initial to the final state.

The cost function C is defined such that it is an aggregate of two constituent cost functions. The first of these is the distance between the final obtained state and the goal state denoted by S; the second is the incremental cost function integrated along the path, indicated by L. Thus, C is given by

$$C = S(Z(T), T) + \int_0^T L(Z(t), u(t), t)\, dt. \quad (5)$$

Minimizing C is also equivalent to minimizing C' which is given by

$$C' = \int_0^T L'(Z(t), u(t), t)\, dt \quad (6)$$

with

$$L' = L + \frac{\partial S}{\partial Z}\dot{Z} + \frac{\partial S}{\partial t} \quad (7)$$

Note that C' is to be minimized under the constraint of the state equations given in (4).

Introducing the Lagrange parameters represented by the vector λ, we recast the problem of minimizing C', defined in (6) into the unconstrained minimization problem of minimizing C'' given by

$$C'' = \int_0^T L' + \lambda^T(f(Z, u, t) - \dot{Z})\, dt \quad (8)$$

where λ is also called the costate or the adjoint state.

Next, we introduce the Hamiltonian H, which is defined as

$$H = L' + \lambda^T f \quad (9)$$

in order to recast the optimization problem in an alternate form.

When the control vector u is unconstrained, i.e. $U_{ad} = \mathbb{R}^m$, the optimal solutions $Z^\times(t)$, $u^\times(t)$, $\lambda^\times(t)$ satisfy the

Euler–Lagrange equations (see [5], [6])

$$\dot{\lambda}^{\times} = -\frac{\partial H}{\partial x}(Z^{\times}, u^{\times}, \lambda^{\times}, t) \tag{10}$$

$$\dot{Z}^{\times} = \frac{\partial H}{\partial \lambda}(Z^{\times}, u^{\times}, \lambda^{\times}, t) \tag{11}$$

$$\frac{\partial H}{\partial u}(Z^{\times}, u^{\times}, \lambda^{\times}, t) = 0. \tag{12}$$

However, when the control vector u is constrained, namely U_{ad} is included in but is not identical to $\mathbb{R}^m$, the algebraic system (12) is no longer valid and this equation is replaced by the maximum principle of Pontryaguin (13):

$$H(Z^{\times}(t), u^{\times}(t), \lambda^{\times}(t), t) \leq H(Z^{\times}(t), u(t), \lambda^{\times}(t), t) \tag{13}$$

for all $u \in U_{ad}$ and at all $t \in [0, T]$.

Equations (10) and (11), which are differential systems of first order and dimension n, determine the evolution of $\lambda^{\times}(t)$ and $Z^{\times}(t)$, and (13) enables one to compute $u^{\times}(t)$ for each t. Equations (10) and (11) contains $2n$ unknowns (n each for the state and the costate) and an additional unknown that is the final time. Thus, we need $2n + 1$ boundary conditions to determine these unknowns. The initial state vector $Z(0)$ provides n initial boundary conditions. The remaining $n + 1$ conditions are obtained from the transversality conditions [5]:

$$\left[\frac{\partial S(Z^{\times}(t), t)}{\partial Z} - \lambda^{\times}(t)\right] = 0 \tag{14}$$

$$\left[H(Z^{\times}(t), u^{\times}(t), \lambda^{\times}(t), t) + \frac{\partial S(Z^{\times}(t), t)}{\partial t}\right]_{t=T} = 0. \tag{15}$$

Equations (10), (11), and (13) through (15) completely define the problem of finding the optimal solution $Z^{\times}$, $u^{\times}$, $\lambda^{\times}$, T.

The problem stated above is a boundary value problem from differential operators, whose solution is more costly than it is for an initial value problem. For this reason, we have opted to derive the solution of the above-mentioned equations using an alternate approach to be described in the following section.

Before concluding this section, we present the special case of minimal time control, for which the state equation is linear. We show below that, in this case, the inequality (13) leads to the solution of the optimal control problem in a straightforward manner.

2.2 *Special Case of Control in Minimal Time with Linear Dependence of the State Equation on the Control Vector*

In this case, the problem is to take the system from an initial state at $t = 0$, to a final state Z_f, in a minimal time T. Thus, the quantity to be minimized is

$$T = \int_0^T dt. \tag{16}$$

Now we consider the special case where the state equation depends linearly on the control vector $u(t)$ as follows:

$$\dot{Z}(t) = \phi(Z(t), t) + B(Z(t), t)u(t) \tag{17}$$

where B is a rectangular $n \times m$ matrix and ϕ is a function not depending on u. Finally, we suppose that the allowable controls are defined by

$$u_{k\min} \leq u_k(t) \leq u_{k\max} \qquad k = \{1, \cdots, m\} \tag{18}$$

In this case, the Hamiltonian H becomes

$$H = 1 + \lambda^T \phi + \lambda^T B u. \tag{19}$$

For this special form of H, (13) yields

$$\lambda^{\times T}(t)B(Z^{\times}(t), t)u^{\times}(t) \leq \lambda^{\times T}(t)B(Z^{\times}(t), t)u(t). \tag{20}$$

Alternatively, if b_k is the kth column of the matrix B, (20) can be written as

$$\sum_{k=1}^{m} (\lambda^{\times T}(t) b_k(Z^{\times}(t), t)) u_k^{\times}(t) \leq \sum_{k=1}^{m} (\lambda^{\times T}(t) b_k(Z^{\times}(t), t)) u_k(t) \tag{21}$$

which for independent u_k's yields

$$u_k^{\times}(t) = \begin{cases} u_{k\min} & \text{if } \lambda^{\times T}(t) b_k(Z^{\times}(t), t) > 0 \\ u_{k\max} & \text{if } \lambda^{\times T}(t) b_k(Z^{\times}(t), t) < 0. \end{cases} \tag{22}$$

Hence, the components $u_k^{\times}$ of the control vector can only assume either their maximum or minimum value. These values change at the switching times when the sign of $\lambda^{\times T}(t) b_k(Z^{\times}(t), t)$ changes. This type of control is refered to as bang-bang [5].

Having described the optimization procedure using the control theory terminology, we go on to show how it applies to the problem at hand, namely the synthesis of multilayer absorbers.

2.3 *Application to the Multilayer Absorber Problem*

We begin by establishing the correspondence between the variables used in the optimal control and the absorber synthesis problems in the following:

(i) the height x above the perfectly conducting plane is identified with the time t in the optimal control problem

(ii) the state vector $Z(x)$ is an n-component vector and the impedance $Z_i(x)$ at height x for the frequency f_i, are the components of this vector. Each $Z_i(x)$ is to be taken from an initial value of 0 at $x = 0$ to a final value as close as possible to the goal state $(1 + j0)$ at the total thickness $x = T$.

(iii) The Riccati equations (3), at frequencies $\{f_1, \cdots, f_i, \cdots, f_n\}$ are the state equations.

The next thing to be specified is the control vector u_k, $k = \{1, \cdots, m\}$. The only possible control parameters are the material parameters $\epsilon(x)$ and $\mu(x)$. At this point, we also introduce an auxiliary function called the "rate" of the material at height x. We can then express $\epsilon(x, f)$ and $\mu(x, f)$ as

$$\epsilon(x, f) = \sum_{k=1}^{m} u_k(x)\epsilon_k(f)$$

$$\mu(x, f) = \sum_{k=1}^{m} u_k(x)\mu_k(f) \tag{23}$$

where $k \in \{1, \cdots, m\}$, m is the number of available materials. ϵ_k and μ_k are the permittivity and the permeability of the kth material respectively.

$$u_k(x) \in [0, 1] \quad \text{and} \quad \sum_{k=1}^{m} u_k(x) = 1 \quad \text{for all } x.$$

With the definition of ϵ and μ given in (23), all points of the convex hull of the material properties, i.e., the envelope of the available materials, can be accessed. For a general choice of u_k, the individual layers can be heterogeneous material. However if one and only one u_k is allowed to be different from 0 (i.e., it takes the value of one) at elevation x, then the material of each layer is purely homogeneous.

Next, we identify the state equation as

$$\frac{dZ_i}{dx} = j\omega_i Z_i^2 \sum_{k=1}^{m} u_k(x)\epsilon_k(f_i) - j\omega_i \sum_{k=1}^{m} u_k\mu_k(f_i) \tag{24}$$

which is a linear function of the control u_k.

We now turn to the discussion of the choice of the cost functions S and L defined in Section 2.1.

For the quantity S, which measures the distance between the final state $(Z_1(T), \cdots, Z_n(T))$ and the goal state $(1, \cdots, 1)$, a possible choice is

$$S = \frac{\alpha}{n} \sum_{i=1}^{n} |Z_i(T) - 1|^2 \tag{25}$$

where α is a parameter chosen by the user to combine the two cost functions, S and $\int L$. These functions are distinctly different in nature, as seen from the definitions given in Section 2.1.

For the quantity L, which measures the cost of the trajectory, we can make either of the following two choices:

(i) $L = 1$ which implies that $\int_0^T L\,dx = T$ and therefore we minimize the thickness of the coating

(ii) $L = \Sigma_1^m \rho_k u_k(x)$ where ρ_k is the volumetric mass of the kth material; for this choice, we minimize the surface mass of the coating.

Now that we have laid the foundations of our optimization procedure, we can proceed to apply it to our synthesis problem. Before we do this, however, we explain in the next section how we can modify the formulation so that it would be better suited to application to our particular problem.

III. Modification of the Optimization Procedure

3.1 Formulation in Terms of a Nested Optimization Problem

In the preceding section, we described how the Pontryagin principle (13) combined with the transversality conditions (14) and (15), and the Euler Lagrange equations (10) and (11) can be used to compute the state vector $Z^\times$, the adjoint state $\lambda^\times$, and the optimal control $u^\times$ to solve our minimization problem. However, the method described above has two drawbacks. First, it calls for the solution of a boundary value problem for the differential system, rather than an initial value problem, and this, in turn, makes the solution procedure computationally intensive. Second, and perhaps the more serious drawback, is that the parameter α must be chosen by the user. A large value of this parameter emphasizes the minimization of the reflection coefficient, whereas a low value focuses on the minimization of the thickness (or of the surface mass) of the coating. It is not an easy task for the user to develop a strategy for choosing α, because the two cost functions are different in nature. In order to circumvent this problem that arises because of the definition of the cost function, which is a combination of two different functions S and $\int L$, we suggest an alternate route, in which we solve a nested minimization problem as explained below.

In this approach, we first define the internal minimization problem, with the cost function $C = \int_0^T L\,dx$, which is a minimal time control problem. We solve it using the algorithm for control in minimal time given in Sec. 2.2, using a prescribed value of the costate vector λ at height $x = 0$, i.e., on the perfectly conducting plane. Equations (10) and (11) yield the evolution of $Z(x)$ and $\lambda(x)$, and (22) yields the value of the control. So, at a given elevation x, $u(x)$, $Z(x)$ and $\lambda(x)$ only depend on the initial value $\lambda(x = 0)$. The total thickness T is defined as the value of x where $S(Z(x))$ reaches its minimum on $[0, X_{\max}]$ where $X_{\max}$ is the maximal allowable thickness. The solution for $S(Z(T))$ thus derived, becomes a function of the initial costate λ at $x = 0$. The next step is to minimize S as a function of this free parameter λ at $x = 0$, which can be viewed as an external minimization problem. This problem will be addressed by using a gradient optimization method described in the next section.

Note that the two minimization problems are nested, because the computation of the cost function of the external problem requires the solution of the internal problem. Note also that the strategy in this method is different from that of the previous section, where the two cost functions were simply combined using the weighting parameter α.

3.2 Non-Convexity of Minimization Problem for S

A convenient way to minimize the cost function S would be to use the gradient method. However, this method is primary suited for convex problems and, hence, it would be necessary for us to modify for our case, which is typically non-convex. For a convex problem, one could start from any point in the λ space and reach the global mini-

mum via the gradient optimization procedure. However, for our problem, we expect in general to reach only a local minimum. Thus, we begin by gridding up the λ space, and we calculate the value of the cost function S for each point of the grid. Then the points of the grid that give the lowest costs S are used as initial values for a gradient method to determine the nearest local minimum. By following this technique we hope to reach the most "desirable" local minima. "Desirable" is defined in terms of minimal cost of the S function. It will be evident from the discussion given in Section V that this hope is reasonable.

For the single frequency case, the λ space is two dimensional, spanning the complex plane $\mathbb{C}$, and it is possible to work with a fine grid without investing an excessive amount of computation time. However, when the minimization problem is solved simultaneously for n frequencies, the λ space is now $2n$ dimensional, i.e. $\mathbb{C}^n$, and we must work with a coarser grid in order to keep the computational time between realistic bounds. It is conceivable that, using such a grid, we will miss some of the local minima that would have yielded better solutions. However, an advantage of the outlined procedure is that we can find several solutions which, while they are not optimal from the point of view of yielding the lowest reflection coefficient, may nevertheless yield thinner screens than that synthetized by the optimal solution. It is also possible that some of these solutions based on local minima yield multilayer solution screens with smaller number of layers than that predicted by the optimal solution, a feature that is desirable from the manufacturing point of view. Using this approach, the user has more flexibility in choosing the design for the screen.

3.3 *Choice for the Cost Function S*

We now discuss the choice of the cost function S to be used in the optimization procedure described above. We have a number of options available for choosing S. Some examples are given below.

(i) One of the simplest choices for the cost function

$$S_1 = \frac{1}{n} \sum_{i=1}^{n} |Z_i(T) - 1|^2$$

which is based on the use of the impedance Z_i at $x = T$.

(ii) A second choice is S_2 which is related to the reflection coefficient:

$$S_2 = \frac{1}{n} \sum_{i=1}^{n} R(f_i)$$

where

$$R(f_i) = 20 \log \left| \frac{Z_i - 1}{Z_i + 1} \right|.$$

However since $R(f)$ is not bounded from below, this form has some drawbacks. Experience shows that using this cost function, we often obtain solutions that have very low reflection coefficient at only one or two of the prescribed frequencies, that are clustered together, and moderately high values for the reflection coefficient at the other frequencies, and yet the value of the cost function S_2 is still reasonably low. These solutions typically have very narrow bandwidth, a feature that is usually not too desirable, because specifications typically call for low reflection coefficient covering a fairly wide band of frequencies.

(iii) To overcome this deficiency, we introduce a new cost function S_3 defined below, that circumvents the narrow bandwidth problem:

$$S_3 = \max_{\{f_1, \cdots, f_n\}} R(f_i)$$

Experience shows that, in general, the use of S_3 yields a better solution than that obtained by S_2.

(iv) Still other types of cost functions can be defined in terms of different reflection coefficient $R_c(f_i)$ for each frequency. The $R_c(f_i)$ are chosen in accordance with specification for RCS reduction, which may be different for different frequencies. Two such cost functions are

$$S_4 = \frac{1}{n} \sum_{i=1}^{n} |R(f_i) - R_c(f_i)|$$

or

$$S_5 = \max_{\{f_1, \cdots, f_n\}} |R(f_i) - R_c(f_i)|$$

(v) One can generalize this type of cost function even further by weighting the norms of the difference $R(f_i) - R_c(f_i)$, which leads to S_6 given by

$$S_6 = \sum_{i=1}^{n} p_i |R(f_i) - R_c(f_i)|$$

with

$$\sum p_i = 1 \qquad p_i > 0.$$

This cost function allows us to accentuate the RCS reduction at certain frequencies prescribed by the user.

IV. Numerical Examples

To illustrate the application of the synthesis procedure for absorbing screens just described in the last section, we present a number of numerical examples in this section. We assumed the availability of approximately 20 different types of materials for the purpose of the numerical calculation below. These were chosen from:

(i) lossless dielectrics with permittivity ϵ between $1 + j0$ and $3000 + j0$.

(ii) lossy dielectrics: ϵ complex varying between $10 + j0$ and $65 + j50$. Fig. 2 depicts the variation of ϵ with frequency for two special materials referred to as DP2 and DF2 in this paper.

(iii) magnetic materials with constant ϵ of $15. + j0$ and a variable μ in the frequency range of interest.

Fig. 2 shows the properties of four materials $F1$, $F4$, MHP1 and MHP2 vs. frequency.

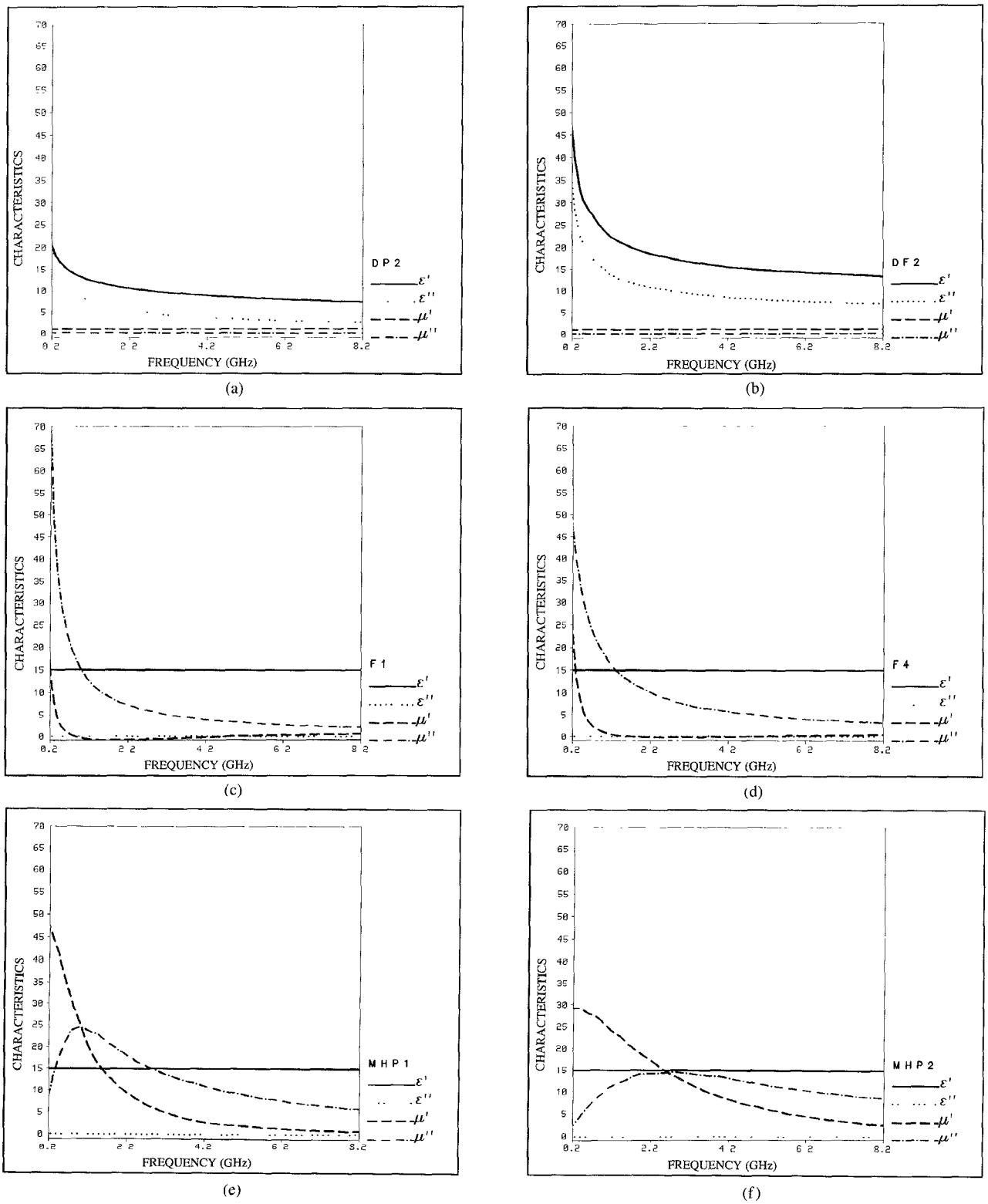


Fig. 2. Characteristics of 6 of the proposed materials.

The optimization was applied in two frequency ranges using the cost function $S3$. The "low frequency" band covered the range 0.2 to 2 GHz while the frequency range was 2 to 8 GHz for the "high frequency" band.

We present both the optimal solution and a few others corresponding to the local minima that yielded screen thicknesses smaller than the optimal one.

4.1 Low Frequency Solutions

The following solutions where derived for the range .2 and 2 GHz

−20 dB solution:	3.187 mm	of	Mat F1
	0.668 mm	of	Mat MHP2
Total thickness:	3.856 mm		

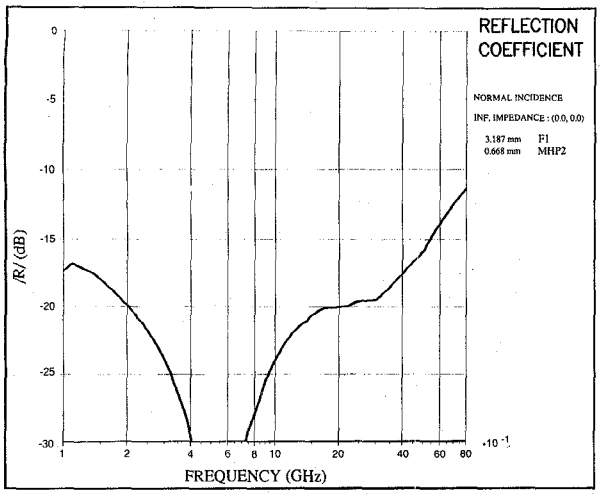


Fig. 3. Low frequency range solution.

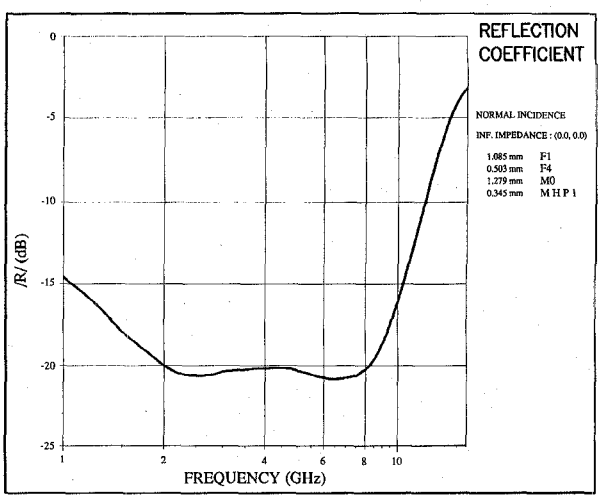


Fig. 4. High frequency range solution.

−19 dB solution:	2.95	of	F1
	0.6	of	MHP2
	0.12	of	F4
Total thickness:	3.67 mm		
−15 dB solution:	2.08	of	F1
	0.272	of	MHP2
	0.409	of	F1
	0.025	of	F4
Total thickness:	2.79 mm		

The plot of the reflection coefficient vs. frequency for the first of these solutions is given in Fig. 3.

4.2 High Frequency Solutions

These are derived in the range 2 and 8 GHz

−20 dB solution:	1.085	of	F1
	0.503	of	F4
	1.279	of	M0
	0.345	of	MHP1
Total thickness:	3.212 mm		
−15 dB solution:	0.121	of	F1
	0.707	of	F4
	0.514	of	M0
	0.413	of	MHP1
Total thickness:	1.755 mm		
−13 dB solution:	0.111	of	F1
	0.7	of	MHP1
	0.129	of	MHP2
Total thickness:	0.94 mm		

The plot of the reflection coefficient versus frequency for the first of these solutions is given in Fig. 4.

V. Comparison of the Simulated Annealing Approach with the Optimal Control Method

In order to reduce the chances that some of the local minima that would be regarded as good solutions to our minimization problem are not overlooked, we have generated a solution to the synthesis problem by using a second method, viz. the Simulated Annealing approach, and have compared the results obtained via using this method and the Optimal Control approach. Simulated Annealing approach is well-suited for application to non-convex optimization problems. In this approach we deviate somewhat from that of Optimal Control method, in that we prescribe a priori the total thickness of the layer. This total thickness is divided into a large number of thin sublayers each of which is only about 50 μm thick. [7] The Simulated Annealing program was run by using the same cost function as given above (S_3), and for the same range of material parameters. It was found that in the low frequency range, viz. between 0.2 to 2 GHz, the solutions obtained by the Simulated Annealing technique were no better than those derived by using the Optimal Control method. However in the high frequency range (2 to 8 GHz) application of the Simulated Annealing method did lead to somewhat lower reflection coefficient for a given total thickness, or, alternatively, slightly thinner screens for the same reflection coefficient. For example, Fig. 5 shows a solution with reflection coefficient of −20 dB for a total thickness of 2.72 mm, which may be compared

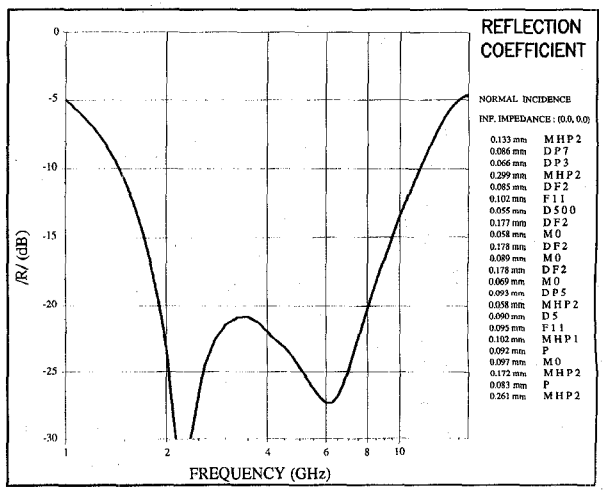


Fig. 5. Solution obtained by Simulated Annealing.

with the 3.21 mm thick solution obtained with the Optimal Control method (see Fig. 4). However, the number of layers required to achieve these results using the Simulated Annealing method is often quite high typically on the order of 20 to 30, and sometimes even more. It would be difficult to manufacture such a coating because of the fragile nature of some of the material and small thicknesses of these layers. In contrast, the Optimal Control method, although it leads only to a local minimum for the cost function, yields results almost always in the realm of practical realizability from the fabrication point of view. Our future plan is to incorporate in the expression for the cost function a penalty function which is biased towards a smaller number of layers, with the hope that the Simulated Annealing approach would also lead to a practically useful type of solution.

VI. Conclusions

The general problem of optimal synthesis of multilayer antireflection coatings is difficult because it requires dealing with a very large number of parameters, e.g. the number of the layers, choice of material media and so on. It is also made complicated by the fact that two quantities, viz., the reflection coefficient and the thickness have to be minimized simultaneously, and that the minimization problem is non-convex.

We have presented an Optimal Control procedure, that provides an effective as well as efficient way of handling this problem. It should be pointed out, however, that user interface is necessary in implementing the procedure, because both the optimal and the locally optimal solutions, generated by the algorithm must be closely examined and evaluated in order to ascertain that they are practical from the manufacturing point of view. The advantage of the present approach over the classical method for designing antireflection coating is, of course, that the solutions obtained are usually much superior than the classical ones, e.g. the Dallenbach screen [8].

Appendix
Oblique Incidence Case

For the oblique incidence case, we consider TE and TM polarizations. The impedance for TE and TM polarizations are respectively denoted by Z_1 and Z_2. Equation (3) for Z is replaced by (26) and (27) for Z_1 and Z_2.

$$\frac{dZ_1}{dx} = j\omega\epsilon\left(1 - \frac{\sin^2\theta}{\epsilon\mu}\right)Z_1^2 - j\omega\mu \qquad (26)$$

$$\frac{dZ_2}{dx} = j\omega\epsilon Z_2^2 - j\omega\mu\left(1 - \frac{\sin^2\theta}{\epsilon\mu}\right) \qquad (27)$$

where θ is the incident angle between the direction of propagation and the outward normal to the surface of the layer.

The goal state, for the case of a single frequency, is now $[1/\cos(\theta)] + j0$ for Z_1 and $\cos(\theta) + j0$ for Z_2. The initial state is $(0 + j0,\ 0 + j0)$.

However, we have as in the normal incident case, an Optimal Control problem in minimal time, and the method outlined in the main text can also be applied to the oblique incidence case.

Furthermore, note that, defining $\epsilon^{\times} = \epsilon(1 - [\sin^2\theta/\epsilon\mu])$ for (26) and $\mu^{\times} = \mu(1 - [\sin^2\theta/\epsilon\mu])$ for (27), (26) and (27) both boil down to (3) with new values of ϵ and μ. But our materials have high index of refraction, thus $\epsilon\mu >> 1$ and $\epsilon^{\times} \simeq \epsilon$, $\mu^{\times} \simeq \mu$. Therefore, the results obtained in the oblique incidence case will not be very different from those of normal incidence.

References

[1] E. F. Knott, J. F. Schaeffer, and M. T. Tuley, *Radar Cross Section*. Norwood, MA: Artech House, 1986, ch. 9.

[2] H. M. Liddell, *Computer-Aided Techniques for the Design of Multilayer Filters*. Bristol: Adam Hilger, 1981.

[3] L. J. Du Toit and J. H. Cloete, "Advances in design of Jaumann absorbers," in *Proc. IEEE-APS URSI Meeting*, Dallas, 1990.

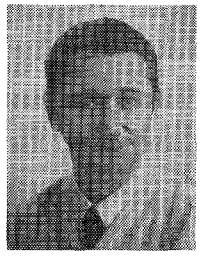
[4] L. Tartar, CEA/LIMEIL, private communication.

[5] J. P. Babary and W. Pelczewski, *Commande Optimale des Systèmes Continus Déterministes*. Paris: Ed Masson, 1985.

[6] L. M. Hocking, *Optimal Control, An Introduction to the Theory with Application*. Oxford: Clarendon, 1991.

[7] B. Souillard, "Application de la méthode de recuit simulé à l'optimisation des systèmes multicouches à fin de discrétion radar," in Internal Rep. under Contract CEA/CESTA no. 91.79.04.

[8] G. Ruck, D. E. Barrick, W. D. Stuart, and C. K. Krichbaum, *R.C.S. Handbook*. New York: Plenum, 1970.



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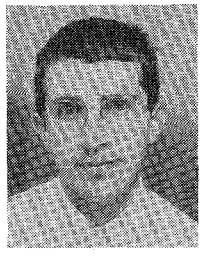

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